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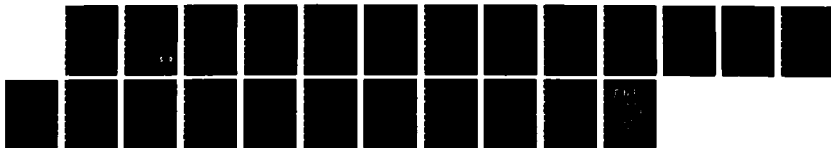
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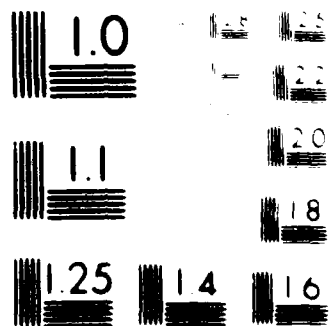
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Coherent States for the Damped Harmonic Oscillator

by

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Prepared for Publication

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Departments of Chemistry and Physics
State University of New York at Buffalo
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Coherent States for the Damped Harmonic Oscillator

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Using the Caldirola-Kanai Hamiltonian for the damped harmonic oscillator, exact coherent states are constructed. These new coherent states satisfy the properties which coherent states should generally have.

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Since the coherent states for the harmonic oscillator have been constructed first by Schrödinger,¹⁾ they have been widely used to describe many fields of physics.²⁾⁻⁵⁾ Recently Nieto and Simmons have constructed coherent states for particles in general potentials⁶⁾ and have applied their formalism to confining one dimensional systems:⁷⁾ harmonic oscillator with centripetal barrier and the symmetric Pöschl-Teller potential, and also to nonconfining one dimensional systems⁸⁾ with the symmetric Rosen-Morse potential and the Morse potential. For the time-dependent systems Lewis and Riesenfeld⁹⁾ have investigated the harmonic oscillator with time-dependent frequency $\omega(t)$. Hartley and Ray¹⁰⁾ has obtained exact coherent states for this time-dependent harmonic oscillator on the basis of Lewis and Riesenfeld theory. Hartley-Ray results satisfy most, but not all, of the properties of the coherent states. In the case of a quantum mechanical model of a damped forced harmonic oscillator, Dodonov and Man'ko¹¹⁾ have introduced the Caldirola-Kanai Hamiltonian¹²⁾ with an external force term and constructed integrals of motion of this Hamiltonian, eigenstates and coherent states. The main flaw of the Dodonov-Man'ko result is its uncertainty relation $\Delta p \cdot \Delta x \geq e^{-\gamma t} \hbar/2$ in which the uncertainty vanishes as $t \rightarrow \infty$. This contradiction is critically reviewed by Greenberger¹³⁾, and Cervero and Villarroel.¹⁴⁾ Greenberger introduced the variable mass: $m = m_0 e^{\gamma t}$ and removed the violation of uncertainty.

In this paper we construct exact coherent states for the damped harmonic oscillator described by the Caldirola-Kanai Hamiltonian

$$\mathcal{H} = e^{-\gamma t} \frac{p^2}{2m} + e^{\gamma t} \frac{1}{2} m \omega_0^2 x^2. \quad (1)$$

We first define ^{the} creation operator a^+ and annihilation operator a , and using these operators we will derive the representations of coherent states and investigate whether our coherent states satisfy the following properties of coherent states: (1) They are eigenstates of annihilation operator. (2) They are created from the vacuum or the ground states by a unitary operator. (3) They represent the minimum uncertainty states. ~~and~~ (4) They are not orthogonal but complete and normalized.

In the preceding paper¹⁵⁾ (hereafter referred to as paper I) we have developed the quantum theory of the damped driven harmonic oscillator with the Caldirola-Kanai Hamiltonian with an external driving force $f(t)$ by path integral method. In paper I, setting $f(t) = 0$, the Hamiltonian is reduced to Eq. (1) and all other results become those corresponding to the Hamiltonian (Eq.(1)), and the Lagrangian, mechanical energy and propagator are given by

$$L = e^{\gamma t} \left(\frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega_0^2 x^2 \right), \quad (2)$$

$$E = e^{-2\gamma t} \frac{p^2}{2m} + \frac{1}{2} m \omega_0^2 x^2, \quad (3)$$

$$K(x, t; x_0, 0) = \left(\frac{m \omega e^{\frac{\gamma}{2} t}}{2\pi i \hbar \sin \omega t} \right)^{\frac{1}{2}} \exp \left[\frac{i m}{4 \hbar} \left(\gamma (x_0^2 - e^{\gamma t} x^2) \right. \right. \\ \left. \left. + \frac{2 \omega}{\sin \omega t} \left((x^2 e^{\gamma t} + x_0^2) \cos \omega t - 2 e^{\frac{\gamma}{2} t} x x_0 \right) \right) \right], \quad (4)$$

with $\omega = (\omega_0^2 - \gamma^2/4)^{\frac{1}{2}}$. Here, the energy expression in Eq. (3) is not equal to the Hamiltonian itself. With the help of Eq. (4) and the wave function of simple harmonic oscillator, we obtain the wave function of the

damped harmonic oscillator:

$$\psi_n(x,t) = \frac{N}{\sqrt{2^n n!}} H_n(Dx) \exp\left(-i\left(n + \frac{1}{2}\right) \cot^{-1}\left(\frac{\gamma}{2\omega} + \cot \omega t\right) - Ax^2\right), \quad (5)$$

where

$$\begin{aligned} N &= \left(\frac{m\omega}{\pi \hbar}\right)^{1/4} \frac{e^{-\frac{1}{2}\gamma t}}{\zeta(t) \sin^{1/2} \omega t}, \\ \zeta^2(t) &= \frac{\gamma^2}{4\omega^2} + \frac{\gamma}{\omega} \cot \omega t + \operatorname{cosec}^2 \omega t, \\ A(t) &= \frac{m\omega}{2\hbar} e^{\gamma t} \left\{ \frac{1}{\zeta(t)^2 \sin^2 \omega t} + i \left(\frac{\gamma}{2\omega} - \cot \omega t + \frac{\frac{\gamma}{2\omega} + \cot \omega t}{\zeta(t)^2 \sin^2 \omega t} \right) \right\}, \\ D(t) &= \left(\frac{m\omega}{\hbar}\right)^{1/4} \frac{e^{\frac{1}{2}\gamma t}}{\zeta(t) \sin \omega t}. \end{aligned} \quad (6)$$

The quantum mechanical expectation values of mechanical energy E (Eq. (3)) take the form

$$\langle E \rangle_{mn} = -\frac{\hbar^2}{2m} e^{-2\gamma t} \left\langle \frac{\partial^2}{\partial x^2} \right\rangle_{mn} + \frac{1}{2} m \omega_0^2 \langle x^2 \rangle_{mn}. \quad (7)$$

The evaluation of Eq. (7) gives the non-zero matrix elements which occur only in the principal diagonal and the two second off-diagonals:

$$\langle E \rangle_{n+2,n} = [(n+2)(n+1)]^{1/2} \theta(t), \quad (8)$$

$$\langle E \rangle_{nn} = \frac{1}{2} \left(n + \frac{1}{2}\right) \hbar \omega e^{-\gamma t} \left[\frac{\omega_0^2}{\omega^2} \zeta(t)^2 \sin^2 \omega t + \frac{1}{\zeta(t)^2 \sin^2 \omega t} \right], \quad (9)$$

where

$$\begin{aligned} \theta(t) = & \frac{1}{4} \hbar \omega e^{-\gamma t} \exp \left\{ \left[2i \cot^{-1} \left(\frac{\gamma}{2\omega} + \cot \omega t \right) \right] \left[f(t)^2 \sin^2 \omega t \right. \right. \\ & - \frac{1}{f(t)^2 \sin^2 \omega t} + \frac{1}{f(t)^2 \sin^2 \omega t} \left\{ \left(\frac{\gamma}{2\omega} - \cot \omega t \right) f(t)^2 \sin^2 \omega t \right. \\ & + \left. \left. \left. \left. \frac{\gamma}{2\omega} + \cot \omega t \right)^2 - 2i \left\{ \left(\frac{\gamma}{2\omega} - \cot \omega t \right) f(t)^2 \sin^2 \omega t + \frac{\gamma}{2\omega} \right. \right. \right. \right. \\ & \left. \left. \left. \left. + \cot \omega t \right\} \right] \right\}. \end{aligned} \quad (10)$$

Taking the complex conjugate and changing n into $(n-2)$ in Eq. (8) we can easily obtain the energy expectation value in $(n-2, n)$ state.

In a similar way to that used to obtain Eq. (7) we can obtain the uncertainty relations in the various states:

$$[(\Delta p)(\Delta x)]_{n+2,n} = \frac{\hbar}{2} \{(n+2)(n+1)\}^{\frac{1}{2}} \beta(t), \quad (11)$$

$$[(\Delta p)(\Delta x)]_{n+1,n} = \frac{\hbar}{2} (n+1) \beta(t), \quad (12)$$

$$[\Delta p \cdot \Delta x]_{nn} = \left(n + \frac{1}{2}\right) \hbar \beta(t), \quad (13)$$

and

$$\beta(t) = \left[1 + \left\{ \left(\frac{1}{8} \left(\frac{\gamma}{\omega} \right)^2 + \left(\frac{\gamma}{\omega} \right) \sin^2 \omega t + \frac{1}{8} \left(\frac{\gamma}{\omega} \right)^2 \sin 2\omega t \right)^2 \right\}^{\frac{1}{2}} \right]. \quad (14)$$

Changing n into $(n-1)$ and $(n-2)$ respectively in Eqs. (12) and (11) we can obtain the uncertainty relations in $(n-1, n)$ and $(n-2, n)$ states.

Before we construct ^{the} annihilation operator a and creation operator a^+ , we give the properties of the coherent states. ^{These} ~~The coherent~~ states can be defined by the eigenstates of the nonhermitian operator a ,

$$a|\alpha\rangle = \alpha|\alpha\rangle. \quad (15)$$

Using the completeness relation for the number representations, we can expand $|\alpha\rangle$ as

$$\begin{aligned} |\alpha\rangle &= e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \\ &= e^{-\frac{1}{2}|\alpha|^2} e^{\alpha a^+} |0\rangle, \end{aligned} \quad (16)$$

where $|0\rangle$ is the vacuum or ground state and is independent of n . The calculation of $\langle\beta|\alpha\rangle$ in Eq. (16) gives

$$\langle\beta|\alpha\rangle = e^{-\frac{1}{2}(|\alpha|^2 + |\beta|^2) + \alpha\beta^*}. \quad (17)$$

Since Eq. (17) has nonzero value for $\alpha \neq \beta$, the states are not orthogonal, but when $|\alpha - \beta|^2 \rightarrow \infty$, the states become orthogonal.

The eigenvalue α of coherent states are complex number $u + iv$, and thus the completeness relation of coherent states is written as

$$\int |\alpha\rangle \langle\alpha| \frac{d^2\alpha}{\pi} = 1, \quad (18)$$

where 1 is the identity operator and $d^2\alpha$ is given by $d(\text{Re}u)d(\text{Im}v)$.

To define a^* and a for the damped harmonic oscillator we make use of Eq. (5) for $\langle x \rangle_{mn}$ and $\langle p \rangle_{mn}$:

$$\begin{aligned}
 \langle x \rangle_{mn} &= \int_{-\infty}^{\infty} \psi_m^*(x) x \psi_n(x) dx \\
 &= \frac{1}{2} (n+1)^{\frac{1}{2}} (\text{Re}A)^{-\frac{1}{2}} \exp\{i \cot^{-1}(\frac{\gamma}{2\omega} + \cot \omega t)\} \delta_{m,n+1} \\
 &\quad + \frac{1}{2} n^{\frac{1}{2}} (\text{Re}A)^{-\frac{1}{2}} \exp\{-i \cot^{-1}(\frac{\gamma}{2\omega} + \cot \omega t)\} \delta_{m,n-1} \\
 &= (n + \frac{1}{2})^{\frac{1}{2}} \mu(t) \delta_{m,n+1} + n^{\frac{1}{2}} \mu^*(t) \delta_{m,n-1}, \quad (19)
 \end{aligned}$$

$$\begin{aligned}
 \langle p \rangle_{mn} &= \int_{-\infty}^{\infty} \psi_m^*(x) \frac{\hbar}{i} \frac{\partial}{\partial x} \psi_n(x) dx \\
 &= i \hbar \sqrt{2} (n+1)^{\frac{1}{2}} \frac{A}{D} \exp\{i \cot^{-1}(\frac{\gamma}{2\omega} + \cot \omega t)\} \delta_{m,n+1} \\
 &\quad + i \hbar \sqrt{2} n^{\frac{1}{2}} (\frac{A}{D} - D) \exp\{-i \cot^{-1}(\frac{\gamma}{2\omega} + \cot \omega t)\} \delta_{m,n-1} \\
 &= (n + \frac{1}{2})^{\frac{1}{2}} \eta(t) \delta_{m,n+1} + n^{\frac{1}{2}} \eta^*(t) \delta_{m,n-1}, \quad (20)
 \end{aligned}$$

where

$$\mu(t) = \frac{1}{2} (\text{Re}A)^{-\frac{1}{2}} \exp[i \cot^{-1}(\frac{\gamma}{2\omega} + \cot \omega t)], \quad (21)$$

$$\eta(t) = \sqrt{2} i \hbar \frac{A}{D} \exp[i \cot^{-1}(\frac{\gamma}{2\omega} + \cot \omega t)], \quad (22)$$

and we have the relation

$$\eta \mu^* - \eta^* \mu = 2i \operatorname{Im} \left\{ \frac{1}{2} (\operatorname{Re} A)^{+1/2} \sqrt{2} i \hbar \frac{\Lambda}{D} \right\} = i \hbar. \quad (23)$$

Therefore, we define ^{the} ~~the~~ annihilation operator a and creation operator a^* for the damped harmonic oscillator as follows

$$a = \frac{1}{i\hbar} (\eta x - \mu p), \quad (24)$$

$$a^* = \frac{1}{i\hbar} (\mu^* p - \eta^* x), \quad (25)$$

where the _{the} expressions of x and p by a and a^* are

$$x = \mu^* a + \mu a^*, \quad (26)$$

$$p = \eta^* a + \eta a^*. \quad (27)$$

Since η is not equal to μ in Eqs. (21) - (22), we can easily conform that a and a^* are not hermitian operators, but the following relations are preserved:

$$[x, p] = i\hbar \quad (28)$$

$$[a, a^*] = 1. \quad (29)$$

Now we evaluate the transformation function $\langle x | \alpha \rangle$ from coherent states to the coordinate representation $|x\rangle$. From Eqs. (15) and (24)

we have

$$(\eta x' - \mu \frac{\hbar}{i} \frac{\partial}{\partial x'}) \langle x' | \alpha \rangle = i \hbar \alpha \langle x' | \alpha \rangle. \quad (30)$$

For convenience we change the variable x' into x and solve this differential equation, ~~and we get~~ ^{to obtain}

$$\langle x | \alpha \rangle = N \exp\left\{ \frac{1}{\mu} \alpha x - (2 i \hbar \mu)^{-1} \eta x^2 \right\}, \quad (31)$$

where N is the integral constant. Taking N to satisfy Eq. (18), we ~~obtain~~ ^{find} the eigenvectors of ~~the~~ ^{the} operator a given in the coordinate representation $|x\rangle$:

$$\langle x | \alpha \rangle = (2 \pi \mu \mu^*)^{-1/2} \exp\left\{ -\frac{1}{2 i \hbar} \frac{\eta}{\mu} x^2 + \frac{\alpha}{\mu} x - \frac{1}{2} |\alpha|^2 - \frac{1}{2} \frac{\mu^*}{\mu} \alpha^2 \right\}. \quad (32)$$

Next we show that a coherent state represents a minimum uncertainty state. With the help of the relations between a , a^\dagger , x and p we evaluate the expectation values of x , p , x^2 and p^2 in state $|\alpha\rangle$:

$$\langle x \rangle = \langle \alpha | \mu^* a + \mu a^\dagger | \alpha \rangle = \mu^* \alpha + \mu \alpha^*,$$

$$\langle p \rangle = \langle \alpha | \eta^* a + \eta a^\dagger | \alpha \rangle = \eta^* \alpha + \eta \alpha^*,$$

$$\langle x^2 \rangle = \mu^{*2} \alpha^2 + \mu \mu^* (1 + 2 \alpha \alpha^*) + \mu^2 \alpha^{*2},$$

$$\langle p^2 \rangle = \eta^{*2} \alpha^2 + \eta \eta^* (1 + 2 \alpha \alpha^*) + \eta^2 \alpha^{*2}.$$

(33)

From Eq. (33) we have

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2 = \mu \mu^*, \quad (34)$$

$$(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2 = \eta \eta^*, \quad (35)$$

and thus the uncertainty relation becomes

$$(\Delta x)(\Delta p) = \{|\eta|^2 |\mu|^2\}^{1/2} = \frac{\hbar}{2} \beta(t). \quad (36)$$

Eq. (36) is the minimum uncertainty corresponding to Eq. (13) in (0,0) state.

All of the formulas, we have derived, are reduced to those of simple harmonic oscillator when $\gamma = 0$. The propagator (Eq. (4)) has very similar form to those of Cheng¹⁵⁾ and others,¹⁷⁾ but the wave function (Eq. (5)) is of new form.

We should note that the same classical equation of motion can be obtained from many different actions, and thus one may have many different propagators corresponding to the actions. Therefore it is very important to get the correct propagator. The mechanical energy (Eq. (3)) is not identical to the Hamiltonian operator (Eq. (1)). ^{Since} ~~Therefore~~ we assume that this Hamiltonian represents the quantum mechanical dissipative system.

^{Figures} Fig. 1 and 2 illustrate the decay of the energy expectation value and the uncertainty relation as a function of γ/ω in (n, n) state. ^{Although} ~~As though~~ we have shown only the principal diagonal element, i.e., $\langle E \rangle_{nn}$ of the energy expectation values, there are four off-diagonals adjacent to the principal diagonal, which are involved in the exponential decaying term

$e^{-\gamma t}$. $\langle E \rangle_{nn}$ approaches ~~the~~ the constant value as $\frac{\gamma}{\omega} \rightarrow 0$. The uncertainty for ^{the} $\Lambda(n, n)$ state with period π (Eq. (13)) is reduced to that of the harmonic oscillator at 0° and 180° .

From all of the above we conclude that the coherent states for the damped harmonic oscillator, with the Caldirola-Kanai Hamiltonian we have constructed, satisfy the properties of coherent states (1)-(4).

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Figure Caption

Fig. 1. Energy expectation value for (n, n) state as a function of ωt at the various value of γ/ω . As γ/ω tends to zero, ^{the} energy approaches ~~the~~ the constant values.

Fig. 2. Uncertainty relation for ^{the} (n, n) state versus ωt at ~~the~~ various values of γ/ω .

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Fig. 1

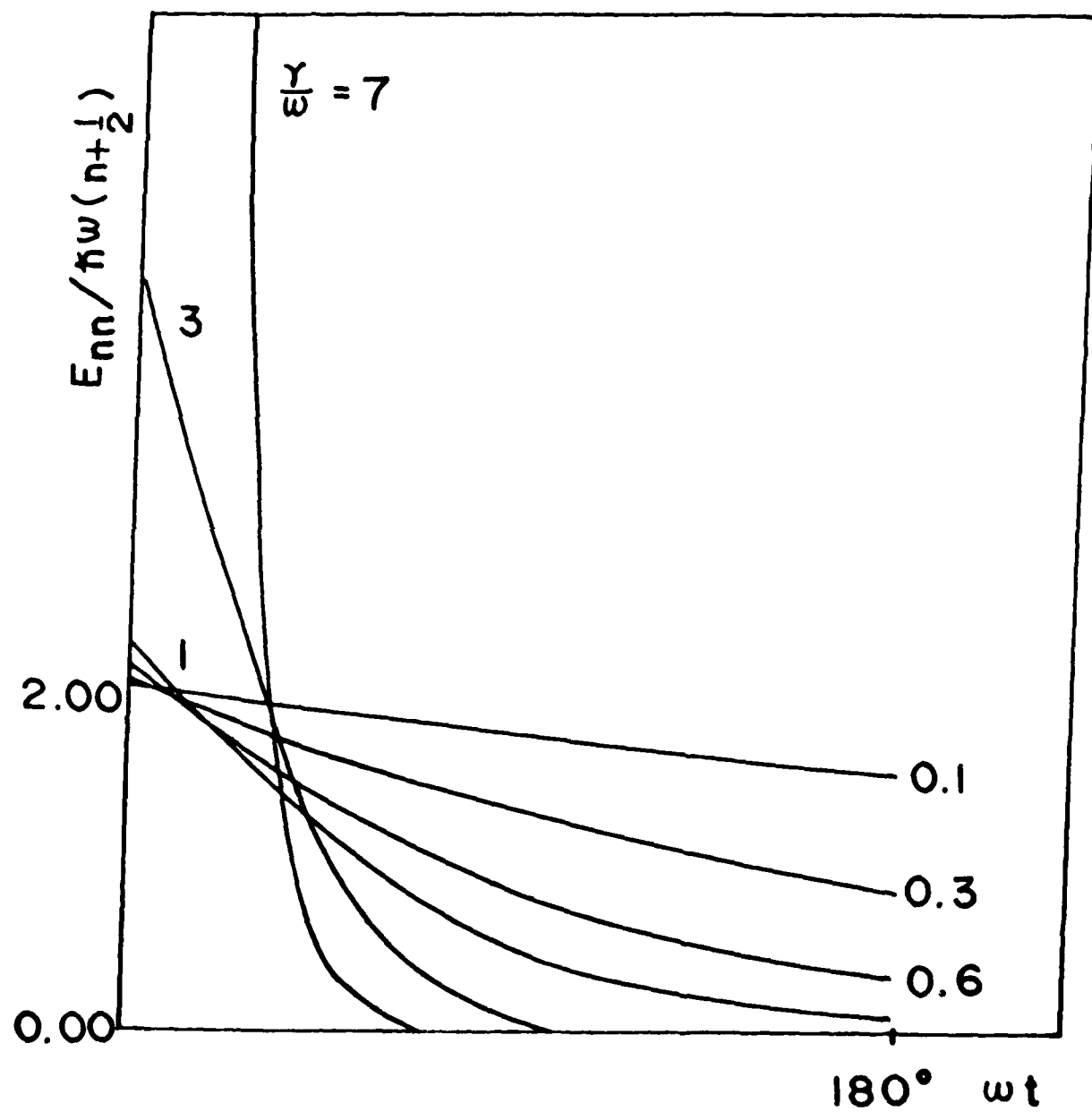
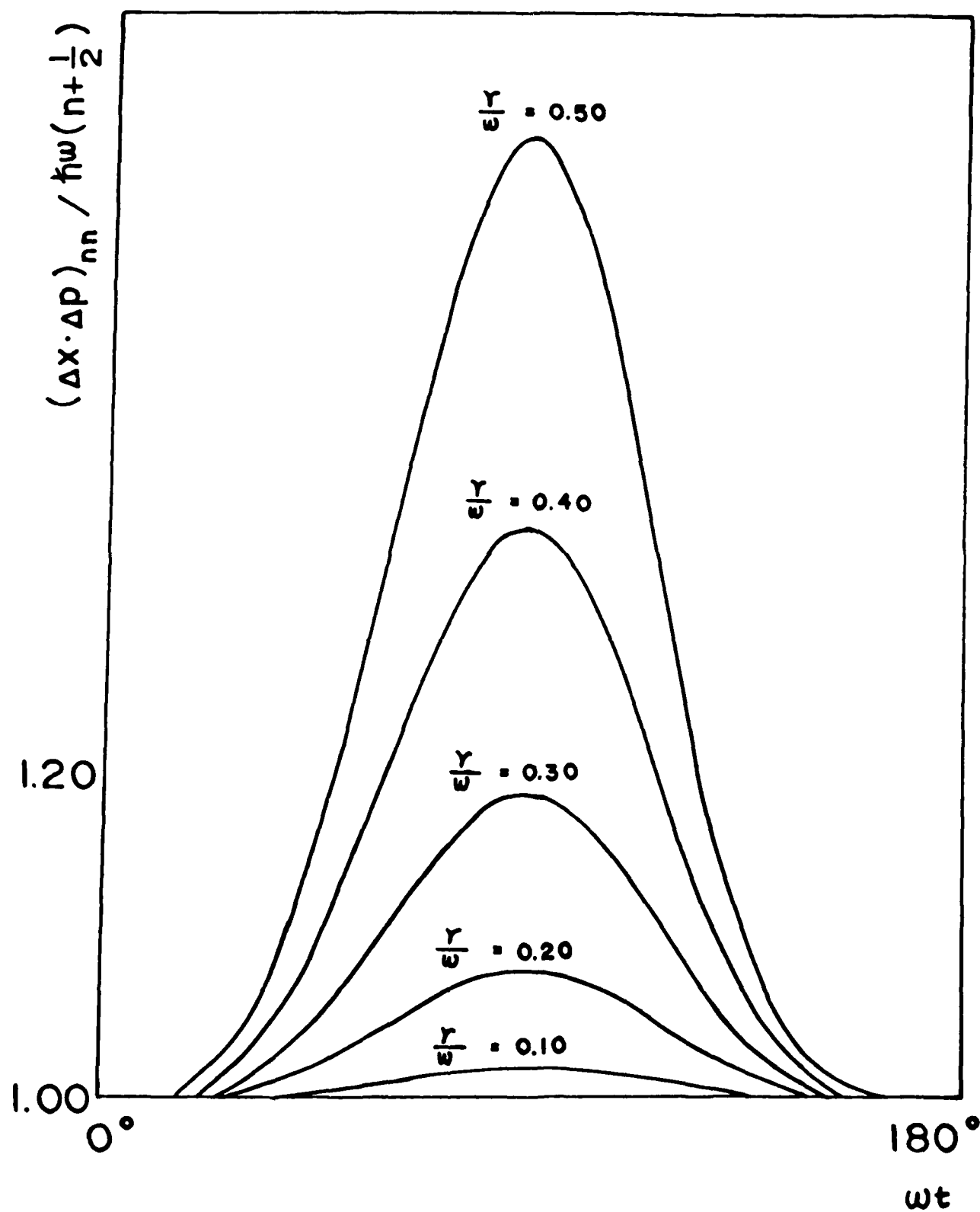


Fig. 2



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